Women in Mathematics Symposium

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Titles and Abstracts

Keynote Address

Asuman Aksoy, Claremont McKenna Colege

Norms That Are Not:
Projections With Respect to Numerical Radii

Concepts used in different areas of mathematics may first appear to be unrelated, but closer examination will uncover many interesting relationships. Operator norm $\|T\|$ of a linear map T and its numerical radius $\nu(T)$ are two such concepts. The standard way to measure the "size" of a linear map is to find its operator norm. However, matrix theory lends a hand to functional analysis and the quadratic form induced by a matrix can be re-defined as a numerical range. Consequently, the radius of the circle centered at 0 enclosing the numerical range gives rise to numerical radius thus enabling us to have another measurement for the "size" of an operator. The interplay between operator norm $\|T\|$ and numerical radius $\nu(T)$ has been subject of much research since Bauer's definition of numerical range in 1960's. After presenting some major results and motivations, seemingly unrelated areas of numerical range and projections, I will show how minimal projections, in particular Fourier projections, can be measured with respect to $\nu(T)$.

Gulhan Bourget, CSU Fullerton
A Modified Test to Detect Significant Genes

Microarray technology has become a very important tool in biological sciences to allow researchers to simultaneously study gene expressions of thousands of genes at once. It is mostly used in drug development. After identifying expressed genes, researchers monitor effects of new drug on subjects to make adjustments to dose levels to reduce or to stop the spread of cancerous cells. The statistical method to identify top 20-100 over-expressed genes is to implement classical t-test. This test requires that observations are independent and there are enough sample sizes. However, in microarray experiments researchers collect 3-5 replicates (small samples), due to microarrays being expensive, and genes can be co-expressed (dependent). Hence, the use of classical t-test is misleading but it is often used. In this talk, I will propose a modified t-test that corrects the problems mentioned above (small sample and dependency of observations).

Maria Isabel Bueno Cachadina, UC Santa Barbara

Recovery of eigenvectors of Matrix Polynomials from Generalized Fiedler linearizations
The standard way to numerically solve polynomial eigenvalue problems is by transforming the matrix polynomial $P(\lambda)$ into a matrix pencil $K(\lambda)$ having the same spectrum (this pencil is known as a linearization of $P(\lambda)$). However, linearizations do not preserve the eigenvectors of the polynomial. Hence, the recovery of eigenvectors of $P(\lambda)$ from the eigenvectors of $K(\lambda)$ becomes a relevant task in the polynomial eigenvalue problem solved by linearization. When the matrix polynomial has a particular structure (symmetric, palindromic, etc) it is desirable, from the numerical point of view, to use linearizations that share this structure. In 2004, Antoniou and Vologiannidis introduced a family of linearizations that was shown to contain linearizations preserving symmetry. From this family, linearizations preserving the palindromic structure have been recently constructed. This makes this kind of pencils a very interesting source of linearizations from the applied point of view. In this talk, we extend the family introduced by Antoniou and Vologiannidis to the family of generalized Fiedler pencils. We also show that there is a simple way to recover the eigenvectors of $P(\lambda)$ from the eigenvectors of any linearization in this family. More precisely, we provide formulas to recover these eigenvectors. Finally, we consider, as particular cases, the linearizations that preserve the symmetric and palindromic structures.

Emek Kose Can, Loyola Marymount
Vector Fields and Differential Forms for Optical Design

A curved mirror realizes a nonlinear transformation, which depends upon the mirror shape. In this talk I will address the problem of determining the mirror shape that will realize a prescribed transformation. The prescribed transformation determines a vector field which should be normal to the sought after mirror, but generally this vector field is not exact. We overcome the limitations of traditional catadioptric systems by constructing the camera projection as well as the mirror surface, using the Frobenius Integration Theorem for differential forms. We will describe several applications including micromirror arrays.

Alona Chubatiuk, USC
Nonparametric Methods For Estimating An Unknown Probability Distribution: Maximum Likelihood and Bayesian Approaches

Suppose we observe a random sample $Y_1, \ldots, Y_N$, where $Y_i \in \mathbb{R}^d$, for all $i = 1, \ldots, N$ and are independent but not necessarily identically distributed. Assume also that the conditional density of Y_i given θ_i is known and denoted by $f(y|\theta_i)$, where the θ_i's are unobserved parameters that are independent and identically distributed given their common but unknown distribution function F. The objective is to estimate F given the data $Y_1, \ldots, Y_N$. We used two different approaches to get the estimates of F: Nonparametric Maximum Likelihood and Nonparametric Bayesian.

Yen Duong, UC Santa Barbara
A New Problem with Expander Graphs

This expositionary talk will describe the problem of explicitly constructing a family of expander graphs, a group of finite, connected, regular graphs with high isoperimetry. I will briefly describe the necessary background in number theory, algebra, and graph theory, then show that a certain Ramanujan graph satisfies the requirement to be an expander graph. We will also describe a few open problems in this subject. Accessible to a general audience.

Cynthia Flores-Monroy, UC Santa Barbara
An Investigation of Compact Manifolds with Positive Curvature

One of the main lines of research in Riemannian Geometry is the study of compact manifolds of positive sectional curvature. Spheres are the simplest examples of such manifolds and differential geometers investigate topological similarities between spheres and positively curved compact manifolds. We present some interactions among the various concepts of curvature and the relatively new concept of isotropic curvature. The results are proved using the Weitzenbock Formula and Hodge Theory.

Julia Galstad, UC Santa Barbara
The Effect of Tilting on theQuivers ofLeft Serial Algebras

Every finite dimensional algebra over an algebraically closed field k is Morita equivalent to a path algebra modulo relations,that is, kQ/I, where Q is a quiver (directed graph) and I is an admissible ideal. The quivers of indecomposable left serial algebras in particular have a nice description. What happens to the quiver when you tilt the algebra? Tan & Koenig (2005) describe theresulting quiver given a few extra hypotheses. This talk will be expository but will be somewhat connected to current research. The talk will emphasize visually pleasing examples and results.

Ellie Grano, UC Santa Barbara
The Jellyfish Algorithm

Algorithms for evaluating closed diagrams are common in topology, e.g. the HOMFLY polynomial and the Kauffman bracket. Stephen Bigelow defined the jellyfish algorithm to evaluate closed diagrams for the ADE planar algebras. I will define the D_{2n} planar algebra, describe the jellyfish algorithm, and show it is well defined. The main result is that the D_{2n} planar algebra is not trivial. This result is known, but the proof is new. Furthermore, this talk is self-contained. We hope to extend these results to more planar algebras in the future.

Johanna Hardin, Pomona College
Outliers when Clustering Microarray Data

Microarray data are well known to be noisy and rife with outliers. The outliers are sometimes interesting in their own right, but often they are simply poor quality measurements that should be removed from the analysis. Unlike many other statistical techniques, clustering methods will always give you cluster outputs regardless of the structure of the data. Though clustering results can be enormously informative, the results can also be misleading if the data have outlying values. In particular, when clustering genes with only tens of samples, a few outlying values can easily change the direction of the relationship between a pair of genes. We propose a metric based on robust distances that gives a more stable clustering containing information about gene co-regulation.

Barbara Herzog, UC Riverside
Toward a Notion of Index for Critical Points of Distance Functions

The index of a smooth function at a critical point is the dimension of the largest subspace on which the Hessian is negative definite. Morse Theory uses critical points (or lack of critical points) of a smooth function as well as index to describe the topology of a space.
In Riemannian geometry distance functions are not smooth meaning that both critical points and the Hessian cannot be defined in the usual way. In 1977 Grove and Shiohama created a definition of critical point for distance functions and used it to describe the topology of a space in the absence of a critical point. This generalization of Morse Theory

has had far reaching consequences. Currently we are working to create a definition of index for distance functions in order to describe the topology of a space at a critical point. A notion of lower index will be presented.

Mihaela Ignatova, USC
Unique continuation and complexity of solutions to higher order elliptic and parabolic equations

We address the strong unique continuation properties of solutions to higher order elliptic and parabolic equations with non-analytic Gevrey coefficients. We provide a quantitative estimate of unique continuation and a new upper bound on the size of the level (nodal) sets of nontrivial solutions with a polynomial dependence on the coefficients. This is a joint work with Igor Kukavica.

Sara Jamshidi, CSU LA
The Sub-Super Solution Method Applied to a Hyperbolic Initial Boundary Value Problem

We are interested in solving the hyperbolic equation

$$\begin{cases} u_{tt} + Au + Fu_t = h & \Omega \times (0,T) \\ u = 0 & \text{on } \partial\Omega \\ u(x,0) = u_t(x,0) = 0 \end{cases}$$

using the sub-super solution method, where A is an symmetric elliptic differential operator and F is a noncoercive Nemytskij operator. We reformulate this as the parabolic problem

$$\begin{cases} w' + ASw + \lambda Bw + Fw = h \\ w_\epsilon(x,0) = 0 \\ w_\epsilon(x,t) = 0 \text{ on } \partial\Omega \end{cases}$$

where $w = u_t$, λ is a parameter and B is a given function. This is then perturbed to the problem

$$\begin{cases} w_\epsilon' + ASw_\epsilon + \epsilon Aw_\epsilon + \lambda Bw_\epsilon + Fw_\epsilon = h \\ w_\epsilon(x,0) = 0 \\ w_\epsilon(x,t) = 0 \text{ on } \partial\Omega. \end{cases}$$

We want to find bounds for w_ϵ and w_ϵ' in the appropriate function spaces.

Gizem Karaali, Pomona College
So what is a Combinatorial Hopf Algebra and what can you do with it?

Hopf algebras were invented to do geometry, but algebraists took off with them and made them theirs. More recently combinatorialists took an active interest in Hopf algebras. So-called combinatorial Hopf algebras have already spawned a new industry around them. The purpose of this talk will be to describe the current state of affairs in this field of research. Basic definitions, motivational examples, and some recent results will be included.

Kathleen Grace Kennedy, UC Santa Barbara
A new algorithm for the Multivariable Alexander Polynomial of a Link

In 1923, Alexander discovered the Alexander Polynomial of a knot, and then in 1970, Conway published a multivariable version of the Alexander polynomial. Last spring, Stephen Bigelow gave a diagrammatic method for calculating the Alexander polynomial of a knot by resolving crossings in a knot or link in a planar algebra. I will present my multivariable version of Stephen Bigelow's calculation, which is the Multivariable Alexander Polynomial defined by Conway. The advantage of this algorithm is that it generalizes to a multivariable tangle invariant.

Hala K. King, Cal Lutheran
An Overview of Two Public Key Cryptosystems

Since the dawn of civilization, people have devised methods to conceal the content of their messages from adversaries. Besides its importance in matters pertaining to military and government intelligence, cryptography is an essential part of today's modern world due to our reliance on computers and the internet. For many years, only private key cryptosystems were used where the sender and recipient of the message shared a secret key that their rivals did not have. The discovery of public key cryptosystems in the 1970's revolutionized the field of cryptography, and the ingenious use of elliptic curves to obtain a new factorization method increased cryptographers' interest in these rich mathematical structures. While designing a course on cryptography, I have researched several encryption systems. This talk will examine the El-Gammal public key cryptosystem and elliptic curve cryptosystems. The security of both systems relies on the difficulty of solving discrete logarithm problems over finite fields.

Olga Korosteleva, CSU Long Beach
Does Extirpation of the Primary Tumor give Boot to Growth of Metastases? Evidence Revealed by Mathmatical Modeling

A model of cancer history is developed to obtain an explicit formula for the distribution of volumes of detectable metastases in a given secondary site at any time post-diagnosis. This model provides a good fit to the volumes of n = 31 bone metastases observed in a breast cancer patient 8 years after diagnosis and removal of the primary tumor. Based on the model, the individual natural history of cancer for the patient is reconstructed. It gives a definitive answer to the following question of major importance in clinical oncology: Does extirpation of the primary breast tumor accelerate the growth of metastases? Specifically, according to the model applied to the patient in question, resection of the primary tumor was followed by a 32-fold increase in the rate of metastases growth.

Nishu Lal, UC Riverside
The Spectral Zeta Function of Fractals and Multivariable Complex Dynamical System

In this talk, we will discuss the spectral zeta function of a self-similar Sturm-Liouville operator on the half real line and C. Sabot's work on connecting the spectrum of this operator with the iteration of a rational map of several complex variables. The Sturm-Liouville operator on $[0, \infty)$ is viewed as a limit of the sequence of operators $\frac{d}{dm_{<n>}}\frac{d}{dx}$ with Dirichlet boundary condition on $I_{<n>} = [0, \alpha^{-n}]$ which are the infinitesimal generators of the Dirichlet form $(a_{<n>}, m_{<n>})$. In particular, it is defined in terms of a self-similar measure m and Dirichlet form a, relative to a suitable iterated function system (IFS) on $I = [0, 1]$. In the case of the Sierpinski gasket, as was shown by A. Teplyaev, extending the known relation by M. Lapidus for fractal strings, the spectral zeta function of the Laplacian has a product structure with respect to the iteration of a rational map on $\mathbb{C}$ which arises from the decimation method. In the case of the above self-similar Sturm-Liouville problem, we obtain an analogous product formula, but now expressed in terms of the (suitably defined) zeta function associated with the dynamics of the corresponding 'renormalization map', viewed as a rational function on $\mathbb{P}^2(\mathbb{C})$. This is joint work with Dr. Lapidus.

Emille Davie Lawrence, Cal Poly Pomona
An Introduction to Right-Angled Artin Groups

The graph of a group is an intuitive way to *visualize* the group. Given a group G, we will define its associated graph, consider some interesting examples, and see why a group's graph can be useful. Conversely, given a simplicial graph, one can define the right-angled Artin group (RAAG) associated to this graph. Furthermore, every RAAG acts geometrically on a CAT(0) cube complex which, in turn, yields some nice results about the group. This talk will be full of examples and pictures.

Arielle Leitner, UC Santa Barbara
Factors Affecting the Academic Success of CSUC Students

What is a better indicator of student success in college: high school GPA or SAT score? In this presentation, we will explore how high school GPA, SAT score, race, gender, major and other factors influenced the academic success of students at Chico State by using data from the 07-08 graduating class.

Kathryn Leonard, CSU Channel Islands
Doing research with academic year undergraduate groups

I will present a brief overview of my general area of research and some student research projects related to it. I will also discuss the ins and outs of mentoring student research groups during the academic year.

Su Liang, CSU San Bernardino
Nail Down the Misconception: $(a + c)/(a + b) = c/b$

Having taught Intermediate Algebra in University of Connecticut for three semesters, I found that a quite number of students kept making a common mistake: $(a + c)/(a + b) = c/b$. This talk will address the reasons of this misconcept and share the teaching approach applied in my classroom. The method - having students work out numerical examples, then discussing the algebra, then looking at the differences between the situations where what the student wants to do is CORRECT versus where it is not, and finally practicing with some more sophisticated examples - is a promising one for other common student errors as well.

Amy Mulgrew, CSU Long Beach
Mathematical Features of Placenta Images Investigated with Techniques of Pattern Recognition

The human placenta nourishes the growing fetus during pregnancy and it seems natural that a healthy placenta would lead to a healthy baby. In a two-part problem, this study aims to (1) extract meaningful features from two-dimensional color images of placentas, and (2) find connections between those features and the medical conditions of both the mother and the child. Techniques such as Principal Component Analysis (PCA) and k-mean clustering have had great success on general object recognition problems. These well-established techniques along with numerous other emerging ones will be utilized in the study of the newly developing field of placenta analysis. Preliminary findings will be presented from which we hope to stimulate a discussion on possible future directions for this new and exciting research area.

Sarah Olson, Tulane
Coupling biochemistry, mechanics, and hydrodynamics to model hyperactivated sperm motility

It is vital for mammalian sperm to achieve hyperactivated motility, characterized by asymmetrical bending and nonlinear trajectories, in order to reach and fertilize the egg. Calcium (Ca2+) dynamics in mammalian sperm are directly linked to hyperactivated motility. These dynamics depend on diffusion, nonlinear fluxes, Ca2+ channels specific to the sperm flagellum, and other signaling molecules. The goal of this work is to couple Ca2+ dynamics to a mechanical model of a motile sperm within a viscous, incompressible fluid. An immersed boundary formulation of regularized Stokeslets is used to investigate the emergent waveforms and velocities. We will present recent progress on elements of this integrative model.

Azadeh Rafizadeh, UC Riverside
Twisted Alexander Polynomials and Fiberability

D. Eisenbud and W. Neumann have developed a theory to determine fiberability of graph links. We use twisted Alexander polynomials to investigate this problem.

Elizabeth Thoren, UC Santa Barbara
How a Fluid Flow is like a Tennis Racket:
An Exploration into the Geometry of Fluid Motion

We may think of the motion of a rigid body (e.g. a tennis racket) as a family of rotations indexed by time. This family of rotations forms a geodesic, or path of least energy, on the manifold of 3-dimensional rotations. Similarly we may think of the motion of a fluid as a family of volume preserving diffeomorphisms of the fluid domain indexed by time. The collection of all volume preserving diffeomorphisms are an algebraic group and an infinite dimensional manifold, thus they form a Lie group. The motion of a fluid is analogous to the motion of a tennis racket in that it can be described by a geodesic on the manifold of all volume preserving diffeomorphisms. I will begin the talk with a description of the Lie group structure of 3-dimensional rotations and then use this example to introduce the Lie group structure of fluid dynamics.

Liming Wang, UC Irvine

A Critical Quantity for Noise Attenuation in Biological Systems

Feedback modules, which appear ubiquitously in biological regulations, are often subject to disturbances from the input, leading to fluctuations in the output. We have identified a critical quantity: the signed activation time that dictates the noise attenuation capability in feedback systems. Our findings suggest that the inverse relationship between the noise amplification rate and the signed activation time could be a general principle for many biological systems regardless of specific regulations or feedback loops.

Laura Zirbel, UC Santa Barbara

Random Embedded Polygons and Characteristics Correlated with Knotting

We will give formulas for average squared end to end distance (the distance between two vertices in a knot) and average squared radius of gyration (how far out the vertices of the polygon gets from the center of mass) over the population of n-edged, non-intersecting, regular polygons embedded in R^3.

Then we will examine some experimental data, including generation method and results connected to the above characteristics.
